

# International Conference on

Partial Differential Equations and Applications

In Honor of 75th Birthday of Prof. Haïm Brezis

July 1-3, 2019 @ Beijing Normal University

▲ June 6<sup>th</sup>, 2016 @BNU

 Venue: The 6<sup>th</sup> Conference Room of Jingshi Hotel
Scientific Committee: Kung-Ching Chang (co-chair), Ta-Tsien Li (co-chair)
Jiguang Bao, Yanyan Li, Yiming Long, Gang Tian
Organizers: Yanyan Li (co-chair), Kaishun Wang (co-chair)

Haigang Li, Zhongwei Tang, Jingang Xiong, Xiaojing Xu

#### Invited Speakers:

Kung-Ching Chang (PKU) Jianchun Chu (Northwestern) Peter Constantin (Princeton) Huabin Ge (Beijing Jiaotong) Changfeng Gui (UT San Antonio) Tianling Jin (HKUST) Ta-Tsien Li (Fudan) Zenghu Li (BNU) Yiming Long (Nankai) Petru Mironescu (U. Lyon) Luc Nguyen (Oxford) Itai Shafrir (Technion)

#### Contact Us:

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#### Supported By:

School of Mathematical Sciences, Beijing Normal University

Yannick Sire (JHU) Gang Tian (PKU) Juan Luis Vázquez (U. Aut. Madrid) Vlad Vicol (NYU) Zhi-Qiang Wang (Utah State) Juncheng Wei (UBC) Jingang Xiong (BNU) Paul Yang (Princeton) Jiangong You (Nankai) Po-Lam Yung (CUHK) Feng Zhou (ECNU)



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**Organizers:** 

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Sponsor:

School of Mathematical Sciences, Beijing Normal University

Beijing Normal University July 1-3, 2019

#### Venue:

Conference Room 6 on the third floor, Jingshi Hotel

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Jianchun Chu (Northwestern)	Gang Tian (PKU)
Huabin Ge (Beijing Jiaotong)	Zhi-Qiang Wang (Utah State)
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Tianling Jin (HKUST)	Jingang Xiong (BNU)
Ta-Tsien Li (Fudan)	Paul Yang (Princeton)
Zenghu Li (BNU)	Jiangong You (Nankai)
Yiming Long (Nankai)	Po-Lam Yung (CUHK)
Petru Mironescu (U. Lyon)	Feng Zhou (ECNU)
Itai Shafrir (Technion)	

## **Registration:**

June 30 11:30-20:30, the lobby of Jingshi Hotel on the first floor July 01 08:00-08:50, Conference Room 6 on the third floor

#### **Contact Info:**

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### Logistics:

Shan Chen, Zixiao Liu, Xuzhou Yang, Binbin Zheng

# Program

July 1, morning		
09:30-09:55	09:30-09:55 Opening ceremony & Conference photo	
10:00-10:45	Nonsmooth critical point theory and applications to the spectral graph theory	Kung-Ching Chang (Peking University)
11:00-11:45	Asymptotic synchronization of second order evolution systems	Ta-Tsien Li (Fudan University)
12:00-14:00	Lunch	

July 1, afternoon		
14:00-14:45	Relative volume comparison along Ricci flow	Gang Tian (Peking University)
14:55-15:40	Minimizers of a variational problem for nematic liquid crystals with variable degree of orientation in two dimensions	Itai Shafrir (Technion)
15:40-16:00	Tea Break	
16:00-16:45	Dynamical System Approach to Spectral Theory of Quasi-Periodic Schrödinger Operators	Jiangong You (Nankai University)
16:55-17:40	On the rigidity and realizations of 3-dimensional hyperbolic polyhedrons	Huabin Ge (Beijing Jiaotong University)
18:00-20:00	Dinner	

July 2, morning		
09:00-09:45	Iteration theories for periodic orbits, old and new	Yiming Long (Nankai University)
10:00-10:45	Borderline asymptotics in Sobolev embedding	Zhi-Qiang Wang (Utah State University)
11:00-11:45	Type II Blow-up for Fujita Equation in the Matano-Merle Exponent Regime	Juncheng Wei (University of British Columbia)
12:00-14:00	Lunch	

July 2, afternoon		
14:00-14:45	Probabilistic models of stochastic population systems	Zenghu Li (Beijing Normal University)
14:55-15:40	C <sup>1,1</sup> regularity of geodesics of singular Kahler metrics.	Jianchun Chu (Northwestern University)
15:40-16:00	Tea Break	
16:00-16:45	New developments in sphere covering inequality	Changfeng Gui (University of Texas at San Antonio)
16:55-17:40	Asymptotic behavior for higher order conformally invariant equations with isolated singularities	Tianling Jin (The Hong Kong University of Science and Technology)
18:00-20:00	Banquet	

July 3, morning		
09:00-09:45	The Sobolev quotient of CR structures in dimension three	Paul Yang (Princeton University)
10:00-10:45	Lifting in Sobolev spaces of manifold-valued maps	Petru Mironescu (Université de Lyon)
11:00-11:45	The Chang-Wilson-Wolff inequality and applications	Po-Lam Yung (The Chinese University of Hong Kong)
12:00-14:00	Lunch	

July 3, afternoon		
14:00-14:45	Half-harmonic maps, minimal surfaces with free boundary and Ginzburg-Landau approximation	Yannick Sire (Johns Hopkins University)
14:55-15:40	Optimal boundary regularity of weak solutions of the fast diffusion equations in domains	Jingang Xiong (Beijing Normal University)
15:40-16:00	Tea Break	
16:00-16:45	Isolated singularities for some nonlinear elliptic equations and applications	Feng Zhou (East China Normal University)
18:00-20:00	Dinner	

# A special talk by Professor Haïm Brezis:

# From Optimal Transport to the Plateau problem via Liquid crystals

**Time:** July 4, 09:00-10:00

**Room:** New Main Building 1124, Beijing Normal University

**Abstract.** I will discuss two proofs of the celebrated Monge-Kantorovich theorem in discrete Optimal Transport (OT). One of them is extremely elementary, self-contained, and can be understood by beginners. I will then describe an application to Liquid Crystals, which provides an explicit formula for the least energy required to produce a configuration with assigned defects. Next I will present striking connections that we recently discovered with P. Mironescu between OT and least area formulas for the classical Plateau problem.

## **Titles and Abstracts**

# Nonsmooth critical point theory and applications to the spectral graph theory

Kung-Ching Chang Peking University

**Abstract:** Existing critical point theories including metric and topological critical point theories are difficult to be applied to some concrete problems in particular polyhedral settings, because the notions of critical sets could be either very vague or too large. To overcome these difficulties, we develop critical point theory for Lipschitzian functions defined on convex polyhedrons incorporated and mixed together with the stratified structure of polytopes and the decomposition of tangent cones. This yields natural extensions of classical results in critical point theory, such as the deformation theorem mountain pass theorem, Liusternik Schnirelmann multiplicity theorem , the Lagrange multiplier theorem, etc. More importantly, eigenvectors for some eigenvalue problems involving graph 1-Laplacian coincide with critical points of the corresponding functions on polytopes, which indicates that the critical point theory proposed in the present paper can be applied to study the nonlinear spectral graph theory.

# C<sup>1,1</sup> regularity of geodesics of singular Kahler metrics

Jianchun Chu Northwestern University

**Abstract:** In this talk, I will first describe a joint work with Valentino Tosatti and Ben Weinkove on optimal  $C^{1,1}$  regularity of geodesics of Kahler metrics. Next, I will show the optimal  $C^{1,1}$  regularity of geodesics of Kahler metrics on compact Kahler varieties away from the singular locus. This is a joint work with Nicholas McCleerey. A key step is to establish the boundary estimate for the complex Monge-Ampere equation that does not require strict positivity of the reference form near the boundary.

# On the rigidity and realizations of 3-dimensional hyperbolic polyhedrons

Huabin Ge Beijing Jiaotong University **Abstract:** The topic is original from Cauchy's rigidity theorem and Steinitz's realization theorem of 3-dimensional Euclidean convex polyhedrons. Cauchy's rigidity theorem says that if two convex polyhedrons P and P'are combinatorically equivalent with corresponding facets being congruent, then P is congruent to P', while Steinitz's realization theorem says that any 3-connected planar graph can be realized as a convex polyhedron.

In this talk, we will give some new results on the rigidity of hyperbolic polyhedrons, we will also build some connections between realizations of hyperbolic polyhedrons and combinatorial Ricci flows on surfaces.

### New developments in sphere covering inequality

Changfeng Gui University of Texas at San Antonio

**Abstract:** In this talk, I will review the Sphere Covering Inequality and present some new forms of the inequality, including a version with singular terms on more general surfaces as well as a dual version. I will also show their applications in the study of singular Liouville-type problems with super harmonic weights. In particular, new uniqueness results for solutions of the singular mean field equation both on spheres and on bounded domains will be discussed. Some new symmetry results for the spherical Onsager vortex equation will also be presented.

# Asymptotic behavior for higher order conformally invariant equations with isolated singularities

Tianling Jin The Hong Kong University of Science and Technology

**Abstract:** We prove sharp blow up rates of solutions of higher order conformally invariant equations in a bounded domain with an isolated singularity, and show the asymptotic radial symmetry of the solutions near the singularity. This is an extension of the celebrated theorem of Caffarelli-Gidas-Spruck for the second order Yamabe equation with isolated singularities to higher order equations. Our approach uses blow up analysis for local integral equations, and is unified for all critical elliptic equations of order smaller than the dimension. We also prove the existence of Fowler solutions to the global equations, and establish a sup\*inf type Harnack inequality of Schoen for integral equations.

#### Asymptotic synchronization of second order evolution systems

Ta-Tsien Li Fudan University

**Abstract:** Under Kalman's criterion and suitable gap condition on the coupling matrices, the unique continiation of a complex system of elliptic operators can be reduced to the observability of a scalar equation. Based on this idea, we establish the strong stability and the asymptotic synchronization by groups for the corresponding evolution systems. This work shows that as the approximate boundary synchronization, Kalman's criterion still plays an important role for the strong stability and the asymptotic synchronization.

#### Probabilistic models of stochastic population systems

Zenghu Li Beijing Normal University

**Abstract:** Branching processes are mathematical models of the evolution of stochastic population systems. The books by Athreya and Ney (1972) and Harris (1963) contain a lot about classical branching processes and their applications. Measure-valued branching processes over abstract spaces were introduced by Watanabe (1968), who proved those processes arose as high-density limits of branching particle systems. The connection of the processes with stochastic evolution equations was first investigated by Dawson (1975). The study has been stimulated from different subjects including stochastic partial differential equations, nonlinear partial differential equations, random trees and graphs. The developments have also led to better understanding of results in those subjects thanks to the works of Aldous (1991, 1993), Dynkin (1994, 2002), Le Gall (1999) and many others. We present a brief review of the study in branching processes focusing on its interplay with stochastic partial differential equations and nonlinear (quasi-linear) partial differential equations.

#### Iteration theories for periodic orbits, old and new

Yiming Long Nankai University

**Abstract:** In this talk, I shall give a survey on the iteration theories for periodic solution orbits of various problems, from 1950s to now with some developments in recent years, including some of their applications to periodic solution problems.

#### Lifting in Sobolev spaces of manifold-valued maps

Petru Mironescu Université de Lyon

**Abstract:** Let  $\Pi: \mathbb{E} \to \mathbb{N}$  be a Riemannian covering map, with  $\mathbb{N}$  compact, and let  $\mathbb{B}^{\mathbb{N}}$  be the unit ball in  $\mathbb{R}^{\mathbb{N}}$ . Given s>0 and  $1 , the lifting problem in the Sobolev space <math>W^{s,p}(\mathbb{B}^{\mathbb{N}},\mathbb{N})$  consists of deciding whether all maps  $u \in W^{s,p}(\mathbb{B}^{\mathbb{N}},\mathbb{N})$  can be lifted in  $W^{s,p}$ , i.e., if we can write every such u in the form  $u = \pi \circ \varphi$ , for some  $\varphi \in W^{s,p}(\mathbb{B}^{\mathbb{N}},\mathbb{E})$ . In general, the answer is negative. We discuss several aspects of this problem: nature of obstructions to lifting, substitutes to lifting, absence of a priori estimates and profile of ``bad'' u's. We also present the complete solution to the lifting problem. Based in part on a joint work with Jean Van Schaftingen, and on previous contributions of Bourgain and Brezis, Bethuel and Chiron.

# Minimizers of a variational problem for nematic liquid crystals with variable degree of orientation in two dimensions

Itai Shafrir *Technion* 

**Abstract:** We study the asymptotic behavior, when  $k \to \infty$ , of the minimizers of the energy  $G_k(u) = \int_{\Omega} ((k-1)|\nabla |u||^2 + |\nabla u|^2)$ , over the class of maps  $u \in H^1(\Omega, \mathbb{R}^2)$  satisfying the boundary condition u = g on  $\partial\Omega$ , where  $\Omega$  is a smooth, bounded and simply connected domain in  $\mathbb{R}^2$  and  $g : \partial\Omega \to S^1$ . The motivation comes from a simplified version of Ericksen model for nematic liquid crystals. We will present similarities and differences with respect to the analog problem for the Ginzburg-Landau energy. This is a joint work with Dmitry Golovaty.

## Half-harmonic maps, minimal surfaces with free boundary and Ginzburg-Landau approximation

Yannick Sire Johns Hopkins University

**Abstract:** I will report on some recent works related to the construction of minimal surfaces with free boundary, using an approach based on half-harmonic maps introduced by Da Lio and Riviere. This approach is rather flexible and gives a new point of view on this old problem both on the elliptic and parabolic sides. I will first describe the regularity theory of half-harmonic maps, motivated both by geometric applications and analytical

developments. I will then move towards the Ginzburg-Landau approximation of those maps, answering a question raised by Da Lio and Riviere in their first work. Tools involve Geometric Measure Theory, Compensation-compactness and a crucial role is played by monotonicity formulae.

#### Relative volume comparison along Ricci flow

Gang Tian Peking University

**Abstract:** In this talk, I will discuss my work with Z.L. Zhang on volume estimates along Ricci flow. We obtained a relative volume comparison result which extends Perelman's famous non-collapsing result and Bishop-Gromov volume comparison for manifolds with Ricci curvature bounded from below.

# Borderline asymptotics in Sobolev embedding

Zhi-Qiang Wang Utah State University

**Abstract:** We report recent work on the asymptotic behavior of extremal functions for the Sobolev embedding from  $H^1$  into  $L^p$  as  $p \rightarrow 2$ . This gives a precise convergence result of the ground states of the scalar field equation, and in turn yields a direct proof of the logarithmic Sobolev inequality by passing limit in the Sobolev embedding. This is a joint work with Chengxiang Zhang.

# Type II Blow-up for Fujita Equation in the Matano-Merle Exponent Regime

Juncheng Wei University of British Columbia

Abstract: I will discuss various Type II blow-up phenomena for the classical

Fujita equation  $u_t = \Delta u + |u|^{p-1}u$  when p belongs to Matano-Merle regime:  $\frac{n+2}{n-2}$ 

# Optimal boundary regularity of weak solutions of the fast diffusion equations in domains

### Jingang Xiong Beijing Normal University

**Abstract:** We show boundary  $C^{2,1}$  regularity and thus optimal higher order regularity for bounded positive weak solutions of the fast diffusion equations in smooth domains. This  $C^{2,1}$  to the boundary regularity problem was raised by Berryman-Holland1980. Previously, the interior smoothness and boundary Holder regularity were proved by Chen-DiBenedetto 1988 and DiBenedetto-Kwong-Vespri 1991. Our proof uses a geometric type structure, namely, a curvature-like evolution equation. This is joint with T. Jin.

# The Sobolev quotient of CR structures in dimension three

Paul Yang Princeton University

**Abstract:** Under the condition that the CR Paneitz operator be non-negative, we were able to prove the positivity of mass when the CR conformal Laplacian is positive. We studied the mass for a family of CR structures called the Rossi sphere, and show that the mass is negative when the structure is sufficiently close to the standard sphere. In addition, we showed that the Sobolev quotient is never attained in this case.

# Dynamical System Approach to Spectral Theory of Quasi-Periodic Schrödinger Operators

Jiangong You Nankai University

**Abstract:** The spectral theory of quasiperiodic operators is a fascinating field which continuously attracts a lot of attentions for its rich background in quantum physics as well as its rich connections with many mathematical theories and methods. In this talk, I will briefly introduce the problems in this field and their connections with dynamical system. I will also talk about some recent results joint with Avila, Ge, Leguil, Zhao and Zhou on both spectrum and spectral measure by reducibility theory in dynamical systems.

# The Chang-Wilson-Wolff inequality and applications

Po-Lam Yung The Chinese University of Hong Kong

**Abstract:** We will discuss an inequality about martingales due to Alice Chang, J.M. Wilson and Tom Wolff, and give some recent applications of the inequality to problems in harmonic analysis involving curvature. Some recent joint work with Shaoming Guo, Joris Roos and Andreas Seeger will be described.

# Isolated singularities for some nonlinear elliptic equations and applications

Feng Zhou East China Normal University

**Abstract:** In this talk, we discuss the isolated singular solutions for nonlinear elliptic equations, in particular for some problems involving Hardy-Leray potential. We present some suitable distributional identity for solutions, including with singularities locating on the boundary. We address some applications on nonexistence of some nonhomogeneous problems with the Hardy-Leray potentials and nonexistence of principle eigenvalue with indefinite potentials. The talk is based on joint works with H.Y. Chen, A. Quaas and R. Peng.